

Geometry of the Space of Triangulations of a Compact Manifold

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Received: 6 October 1994 / Accepted: 1 February 1996

Abstract: In this paper we study the space T_M of triangulations of an arbitrary compact manifold M of dimension greater than or equal to four. This space can be endowed with the metric defined as the minimal number of bistellar operations required to transform one of two considered triangulations into the other. Recently, this space became an object of study in Quantum Gravity because it can be regarded as a “toy” discrete model of the space of Riemannian structures on M .

Our main result can be informally explained as follows: Let M be either any compact manifold of dimension greater than four or any compact four-dimensional manifold from a certain class described in the paper. We prove that for a certain constant $C > 1$ depending only on the dimension of M and for all sufficiently large N the subset $T_M(N)$ of T_M formed by all triangulations of M with $\leq N$ simplices can be represented as the union of at least $[C^N]$ disjoint non-empty subsets such that any two of these subsets are “very far” from each other in the metric of T_M . As a corollary, we show that for any functional from a very wide class of functionals on T_M the number of its “deep” local minima in $T_M(N)$ grows at least exponentially with N , when $N \rightarrow \infty$.

0. Introduction

Let M be a compact PL-manifold, T_M be the (discrete) set of all triangulations of M . (By a “triangulation of M ” we mean in this paper a simplicial complex such that its space is PL-homeomorphic to M . We do not distinguish between simplicially isomorphic triangulations and regard them as identical.) There are many ways to introduce a natural metric on T_M . For example, the results of Pachner imply that any triangulation of M can be transformed into any other triangulation of M by a finite sequence of bistellar operations ([P1, P2]). (A bistellar operation can be defined as follows. Let n denote the dimension of M . Consider a subcomplex K

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This work was partially supported by the New York University Research Challenge Fund Grant, by Grant ARO-DAAL-03-92-G-0143 and by NSERC Grant OGP0155879