Yangians, Integrable Quantum Systems and Dorey's Rule

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Abstract: It was pointed out by P. Dorey that the three-point couplings between the quantum particles in affine Toda field theories have a remarkable Lie-theoretic interpretation. It is also well known that such theories admit quantum affine algebras as "quantum symmetry groups," and widely believed that the quantum particles correspond to the so-called fundamental representations of these algebras. This led to the conjecture that Dorey's rule should describe when a fundamental representation occurs with non-zero multiplicity in a tensor product of two other fundamental representations. The purpose of this paper is to prove this conjecture, both for quantum affine algebras and for Yangians. The result reveals a hitherto unsuspected role played by Coxeter elements (and their twisted analogues) in the representation theory of these algebras.

1. Introduction

Quantum groups arose from the quantum inverse scattering method, developed by the Leningrad school [13] to solve integrable quantum systems. They provide, in particular, a way to understand the solutions of the quantum Yang–Baxter equation (R-matrices) associated to such systems, and a general framework for producing new solutions. Of special importance are the solutions which depend on a complex ("spectral") parameter; those which are rational, or trigonometric, functions of this parameter arise from the quantum groups called Yangians, or quantum affine algebras, respectively (see [11, 12], and Chapter 12 in [8] for background information).

More recently, quantum groups have arisen in another guise in connection with $1 + 1$ dimensional integrable quantum field theories, namely as the algebras satisfied by certain non-local conserved currents. For example, Yangians appear as "quantum symmetry algebras" in G-invariant Wess–Zumino–Witten models [1], while quantum affine algebras appear in affine Toda field theories (ATFTs) [2]. In [10], Dorey gave a remarkable Lie-theoretic description of the classical three-point couplings (or "fusings") in certain integrable field theories, including ATFTs. It is the purpose of this paper to interpret Dorey's rule in terms of the representation theory of Yangians and quantum affine algebras.