Commun Math Phys 181, 205 – 263 (1996)

Communications in Mathematical Physics

Hecke Algebras at Roots of Unity and Crystal Bases of Quantum Affine Algebras*

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Received: 10 November 1995 / Accepted: 21 February 1996

Abstract: We present a fast algorithm for computing the global crystal basis of the basic $U_q(\widehat{\mathfrak{sl}}_n)$-module. This algorithm is based on combinatorial techniques which have been developed for dealing with modular representations of symmetric groups, and more generally with representations of Hecke algebras of type A at roots of unity. We conjecture that, upon specialization $q \to 1$, our algorithm computes the decomposition matrices of all Hecke algebras at a n^{th} root of 1.

1. Introduction

This article arose as an attempt to understand the extensive numerical tables published by James in the appendix of [14]. These tables contain some decomposition matrices of q-Schur algebras at roots of unity, which contain as submatrices the decomposition matrices of Hecke algebras.

A major problem in the p-modular representation theory of the symmetric group $\mathfrak{S}_m$ is to find an algorithm to compute its decomposition matrix. As a matter of fact, this matrix is naturally indexed by the same labels as the decomposition matrix of the Hecke algebra $H_m(\zeta)$, where ζ is a complex p^{th} root of unity. As observed by James, these two matrices are very close to each other, although not equal in general, and apart from its independent interest, the computation of the decomposition matrices of Hecke algebras at roots of unity should be considered as an important step towards the modular case.

The decomposition matrix of $H_m(\zeta)$ is in principle expressible in terms of specializations at 1 of certain (inverse) affine Kazhdan–Lusztig polynomials. Unfortunately, these polynomials do not seem to be easily computable. The aim of this paper is to present a fast conjectural algorithm for calculating the decomposition matrices of Hecke algebras at roots of unity.

This algorithm was actually designed for solving a quite different problem, namely, to compute in an efficient way Kashiwara's global crystal basis of the

* Partially supported by PRC Math-Info and EEC grant n^0 ERBCHRXCT930400