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White Noise Perturbation of the Viscous Shock Fronts of the Burgers Equation

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Abstract: We study the front dynamics of solutions of the initial value problem of the Burgers equation with initial data being the viscous shock front plus the white noise perturbation. In the sense of distribution, the solutions propagate with the same speed as the unperturbed front, however, the front location is random and satisfies a central limit theorem with the variance proportional to the time t, as t goes to infinity. With probability arbitrarily close to one, the front width is $O(1)$ for large time.

1. Introduction

We are concerned with the initial value problem of the Burgers equation:

$$u_t + uu_x = \nu u_{xx}, \quad \nu > 0,\ x \in R^1, \tag{1.1}$$

with initial data:

$$u(x,0) = u_s + V_x, \tag{1.2}$$

where $u_s = u_s(x)$ is the profile of the viscous shock front connecting one and zero, V_x is the white noise, or formally the derivative of a two-sided Wiener process W_x starting from zero. Without the white noise perturbation, we have the exact solution:

$$u(x,t) = \left(1 + \exp\left\{\frac{1}{2\nu}\left(x - \frac{1}{2}t\right)\right\}\right)^{-1}, \tag{1.3}$$

where x can be shifted by any constant $x_0 \in R^1$, and we choose it to be zero for convenience. It is well known that the viscous shock front (1.3) is asymptotically stable if it is perturbed by an integrable function at $t = 0$, see Ilin and Oleinik [8]. We are interested here in the behavior of viscous shock fronts under random perturbations. In reality, random perturbations abound in dissipative dynamical systems admitting front solutions. In the case of conservative systems derived based on conservation of mass, through suitable asymptotic reductions, one often ends up with a scalar conservation law with either random coefficients or