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Morse Homotopy and Chern–Simons Perturbation Theory

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Abstract: We define an invariant of a three manifold equipped with a flat bundle with vanishing homology. The construction is based on Morse theory using several Morse functions simultaneously and is regarded as a higher loop analogue of various product operations in algebraic topology. There is a heuristic argument that this invariant is related to perturbative Chern–Simons Gauge theory by Axelrod–Singer, etc. There is also a theorem which gives a relation of the construction to open string theory on the cotangent bundle.

Contents

0. Introduction

Let M be a $2n$ dimensional manifold and N be its n dimensional submanifold. We consider a current T_N such that $T_N(\omega) = \int_N \omega$. We try to justify the integral

$$\int_M T_N \wedge T_N \,. \tag{0.1}$$

(We remark that $T_N \wedge T_N$ itself is not well defined.) One way to do so is to take a perturbation N' of N so that N' and N are transversal to each other and consider

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