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The Conformal Spin and Statistics Theorem

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Dedicated to Daniel Kastler on the occasion of his seventieth birthday

Abstract: We prove the equality between the statistics phase and the conformal univalence for a superselection sector with finite index in Conformal Quantum Field Theory on S^1. A relevant point is the description of the PCT symmetry and the construction of the global conjugate charge.

Introduction

During recent years Conformal Quantum Field Theory has become a widely studied topic, especially on low dimensional space-times because of physical motivations such as the desire of a better understanding of two-dimensional critical phenomena, and also for its rich mathematical structure providing remarkable connections with different areas such as Hopf algebras, low dimensional topology, knot invariants and subfactors, among many others.

The Operator Algebra approach furnishes a powerful tool of investigation in this context, not only because it naturally leads to a model independent and intrinsic analysis, focusing on essential aspects such as the relative position of the local von Neumann algebras, but also because it makes visible otherwise hidden natural structures yielding results inaccessible by different methods.

Two examples of this kind, the geometric description of the Tomita–Takesaki modular structure of the local von Neumann algebras [1, 18, 4], and the connection of the statistics of a superselection sector with the Jones index theory of subfactors [20], will play a fundamental role in this paper. These methods are present and important in general Quantum Field Theory, but provide an even richer structure in the low-dimensional case, conformal theories on S^1 in particular.

In the early seventies Doplicher, Haag and Roberts [7, 8] developed a theory of superselection sectors, in the sense of [31], in the algebraic framework proposed by Haag and Kastler [17] starting from first principles. They described a superselection sector by a localized endomorphism ρ of the C^*-algebra generated by the local

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