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A Central Limit Theorem for the Fourth Wick Power of the Free Lattice Field

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Abstract: Let G_a be the free lattice field measure of mass m_0 on aZ^d, and $:\phi_x^4:$ be the corresponding fourth Wick power of the lattice field ϕ_x. Let $g \in C_0(R^d)$, $g \geqq 0$, be a given function and $a' = a'(a) \geqq a$ satisfy: $\lim_{a\to 0^+} a' = 0$ and $a'Z^d \subset aZ^d$. We prove that if $d \geqq 3$, or $d = 2$ and $\lim_{a\to 0^+} a'|\log a|^2 = \infty$, then $\{a'^d \sum_{x\in a'Z^d} g_x :\phi_x^4:\}$ satisfies the central limit theorem: there is $V(a,a')$ with $\lim_{a\to 0^+} V(a,a') = \infty$ such that the distribution of $V(a,a')^{-1} a'^d \sum_{x\in a'Z^d} g_x :\phi_x^4:$ under G_a is convergent to the standard normal distribution, as $a \to 0^+$.

1. Introduction

Let G_a be the free lattice field measure of mass $m_0 > 0$ and lattice spacing $a > 0$ on $aZ^d = \{ax : x \in Z^d\}$, and let $\langle \cdot \rangle_{G_a}$ denote the expectation with respect to G_a. Let

$$C^{(a)}(x-y) = \langle \phi_x \phi_y \rangle_{G_a}.$$

G_a is thus the (lattice) Gaussian measure with covariance $C^{(a)}$. It is easy to show that (see [Si, BFS])

$$C^{(a)}(x-y) = (2\pi)^{-d} \int_{[-\frac{\pi}{a},\frac{\pi}{a}]^d} \left[m_0^2 + 2a^{-2} \sum_{j=1}^{d} (1 - \cos ak_j) \right]^{-1} e^{ik\cdot(x-y)} dk\,,$$

with $k = (k_1, \ldots, k_d)$. Let $:\phi_x^4:$ be the fourth Wick order of ϕ_x, i.e.

$$:\phi_x^4: = \phi_x^4 - 6\phi_x^2 \langle \phi_x^2 \rangle_{G_a} + 3\langle \phi_x^2 \rangle_{G_a}^2\,.$$

Let $g(\geqq 0) \in C_0(R^d)$ be a given function and $a' = a'(a)$ satisfy: $a'Z^d \subset aZ^d$ and $\lim_{a\to 0^+} a' = 0$. From this we can see that $a' \geqq a$. For simplicity we also assume that $\lim_{a\to 0^+} \frac{a}{a'}$ exists. The main aim of this paper is to discuss conditions on a and a' such that the central limit theorem holds for the system $\{\xi(a,a')\}$, where

$$\xi(a,a') = a'^d \sum_{x\in a'Z^d} g_x :\phi_x^4:\,.$$