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Deformation Quantizations With Separation of Variables on a Kähler Manifold

Alexander V. Karabegov
Joint Institute for Nuclear Research, Dubna 141980, Russia
E-mail: karabegov@main1.jinr.dubna.su

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Abstract: We give a simple geometric description of all formal differentiable deformation quantizations on a Kähler manifold M such that for each open subset $U \subset M$ $\star$-multiplication from the left by a holomorphic function and from the right by an antiholomorphic function on U coincides with the pointwise multiplication by these functions. We show that these quantizations are in 1–1 correspondence with the formal deformations of the original Kähler metrics on M.

1. Introduction

Formal deformation quantization on a symplectic manifold M (see [1]) is a structure of associative algebra on the space of formal series $C^\infty(M)[[\nu]]$ such that the multiplication in this algebra (denoted by $\star$ and named $\star$-multiplication) is a deformation of the point-wise product of functions on M and the commutator corresponding to the $\star$-multiplication is a deformation of the Poisson bracket $\{\cdot,\cdot\}$ on M.

Deformation quantization is called differentiable if the $\star$-product is given by a formal series of bidifferential operators. Differentiable $\star$-product can be restricted to any open subset $U \subset M$.

The formal product $\star$ can be thought of as an asymptotic expansion in a parameter $\hbar$ of some hypothetical family of noncommutative associative products $\{*_\hbar\}$ of $\hbar$-dependent operator symbols on M such as Weyl symbols, Wick and anti-Wick symbols or their generalizations for curved phase spaces (see [2]). These symbol products have to satisfy the following correspondence principle. For the functions φ, ψ on M $\varphi *_\hbar \psi \to \varphi\psi$ and $\hbar^{-1}(\varphi *_\hbar \psi - \psi *_\hbar \varphi) \to i\{\varphi, \psi\}$ as $\hbar \to 0$.

Only a very limited number of examples is known where deformation quantization appears in such a way from some concrete family of symbol products $\{*_\hbar\}$.

Berezin's quantization on Kähler manifolds (see [2]) provides important examples of differentiable deformation quantizations via the asymptotic expansion of the product of covariant symbols. Such deformation quantizations were obtained on the orbits of compact semisimple Lie groups in [7 and 3] and on bounded symmetric domains in [6 and 4].