Simple Currents and Extensions of Vertex Operator Algebras

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Abstract: We consider how a vertex operator algebra can be extended to an abelian intertwining algebra by a family of weak twisted modules which are *simple currents* associated with semisimple weight one primary vectors. In the case that the extension is again a vertex operator algebra, the rationality of the extended algebra is discussed. These results are applied to affine Kac–Moody algebras in order to construct all the simple currents explicitly (except for E_8) and to get various extensions of the vertex operator algebras associated with integrable representations.

1. Introduction

Introduced in [B] and [FLM], *vertex operator algebras* are essentially *chiral algebras* as formulated in [BPZ] and [MoS], and provide a powerful algebraic tool for studying the general structure of conformal field theory. For a vertex operator algebra V, one wishes to adjoin certain simple V-modules to get a larger algebraic structure so that certain data such as fusion rules and braiding matrices are naturally incorporated. The introduction of the notions of *generalized vertex (operator) algebra* and *abelian intertwining algebra* in [DL] was made in this spirit. A similar notion called vertex operator *para-algebra* was independently introduced and studied in [FFR] with different motivations. Also see [M].

In this paper, we study how a vertex operator algebra can be extended to an abelian intertwining algebra by a family of weak twisted modules which are *simple currents* associated with semisimple weight one primary vectors. In the case that the extension is again a vertex operator algebra, we discuss the rationality of the extended algebra. Applying these results to affine Kac–Moody algebras we construct all the simple currents explicitly (except for E_8) and get various extensions of the vertex operator algebras associated with integrable representations.