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The Wigner Semi-Circle Law in Quantum Electro Dynamics

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Abstract: In the present paper, the basic ideas of the *stochastic limit of quantum theory* are applied to quantum electro-dynamics. This naturally leads to the study of a new type of quantum stochastic calculus on a *Hilbert module.* Our main result is that in the weak coupling limit of a system composed of a free particle (electron, atom,...) interacting, via the minimal coupling, with the quantum electromagnetic field, a new type of quantum noise arises, living on a Hilbert module rather than a Hilbert space. Moreover we prove that the vacuum distribution of the limiting field operator is not Gaussian, as usual, but a nonlinear deformation of the Wigner semi-circle law. A third new object arising from the present theory, is the so-called *interacting Fock space.* A kind of Fock space in which the n quanta, in the n-particle space, are not independent, but interact. The origin of all these new features is that we do not introduce the dipole approximation, but we keep the exponential response term, coupling the electron to the quantum electromagnetic field. This produces a nonlinear interaction among all the modes of the limit master field (quantum noise) whose explicit expression, that we find, can be considered as a nonlinear generalization of the *Fermi golden rule.*

0. Introduction

Quantum electro-dynamics (QED) studies the interaction between matter and radiation. Due to the nonlinearity of this interaction (cf. (1.2) below), an explicit solution of the equations of motion is not known and, for their study, several types of approximations have been introduced.

Probably the best known of these approximations is the *dipole approximation* in which the so-called *response term* ($e^{ik\cdot q}$ in (1.2)) which couples matter, represented by an electron in position q, to the k^{th} mode of the EM-field, is assumed to be 1. The dipole approximation has its physical motivations in the fact that, at optical frequencies and for atomic dimensions one estimates that $k\cdot q \approx 10^{-3}$ (cf. [10]) and therefore very small. Most of the concrete applications of QED (e.g. in quantum optics, laser theory, ...) have been obtained under the dipole