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Holomorphic Bundles and Many-Body Systems

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Abstract: We show that spin generalization of elliptic Calogero–Moser system, elliptic extension of Gaudin model and their cousins are the degenerations of Hitchin systems. Applications to the constructions of integrals of motion, angle-action variables and quantum systems are discussed. The constructions of classical systems are motivated by Conformal Field Theory, and their quantum counterparts can be thought of as being the degenerations of the critical level Knizhnik–Zamolodchikov–Bernard equations.

1. Introduction

Integrable many-body systems attract attention for the following reasons: they are important in condensed matter physics and they appear quite often in two dimensional gauge theories as well as in conformal field theory. Recently they have been recognized in four dimensional gauge theories.

Among these systems the following ones will be of special interest for us:

1. *Spin generalization of Elliptic Calogero–Moser model* – it describes the system of particles in one (complex) dimension, interacting through the pair-wise potential. The explicit form of the Hamiltonian is:

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2} + \sum_{i \neq j} \mathrm{Tr}\,(S_i S_j)\wp(z_i - z_j)\,,$$

where z_i are the positions of the particles, p_i – corresponding momenta and S_i are the "spins" – $l \times l$ matrices, acting in some auxiliary space. The conditions on S_i will be specified later. The only point to be mentioned is that the Poisson brackets between p, z, S are the following:

$$\{p_i, z_j\} = \delta_{ij}\,,$$

$$\{(S_i)_{ab}, (S_j)_{cd}\} = \delta_{ij}(\delta_{ad}(S_i)_{bc} - \delta_{bc}(S_i)_{ad})\,.$$