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Theory of Tensor Invariants of Integrable Hamiltonian Systems. I. Incompatible Poisson Structures

Oleg I. Bogoyavlenskij*

Department of Mathematics and Statistics, Queen's University, Kingston, Canada, K7L 3N6

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Abstract: This paper develops a new theory of tensor invariants of a completely integrable non-degenerate Hamiltonian system on a smooth manifold M^n. The central objects in this theory are supplementary invariant Poisson structures P_c which are incompatible with the original Poisson structure P_1 for this Hamiltonian system. A complete classification of invariant Poisson structures is derived in a neighbourhood of an invariant toroidal domain. This classification resolves the well-known Inverse Problem that was brought into prominence by Magri's 1978 paper devoted to the theory of compatible Poisson structures. Applications connected with the KAM theory, with the Kepler problem, with the basic integrable problem of celestial mechanics, and with the harmonic oscillator are pointed out. A cohomology is defined for dynamical systems on smooth manifolds. The physically motivated concepts of dynamical compatibility and strong dynamical compatibility of pairs of Poisson structures are introduced to study the diversity of pairs of Poisson structures incompatible in Magri's sense. It is proved that if a dynamical system V preserves two strongly dynamically compatible Poisson structures P_1 and P_2 in a general position then this system is completely integrable. Such a system V generates a hierarchy of integrable dynamical systems which in general are not Hamiltonian neither with respect to P_1 nor with respect to P_2. Necessary conditions for dynamical compatibility and for strong dynamical compatibility are derived which connect these global properties with new local invariants of an arbitrary pair of incompatible Poisson structures.

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