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On the Continuity of the Critical Value for Long Range Percolation in the Exponential Case

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Abstract: We show that for a long range percolation model with exponentially decaying connections, the limit of critical values of any sequence of long range percolation models approaching the original model from below is the critical value for the original long range percolation model As an interesting corollary, this implies that if a long range percolation model with exponential connections is supercritical, then it still percolates even if all long bonds are removed We also show that the percolation probability is continuous (in a certain sense) in the supercritical regime for long range percolation models with exponential connections.

1. Introduction

The purpose of this paper is to investigate the continuity from below of the critical value for long range percolation models We begin with a description of long range percolation We let $\mathbf{Z}^d$ denote the standard d-dimensional cubic lattice with the usual L_1 norm given by $|(x_1,x_2, \ldots ,x_d)| = \sum_{i=1}^{d} |x_i|$ If S and T are subsets of $\mathbf{Z}^d$, we let $d(S,T) = \min_{s\in S, t\in T} |s-t|$ be the L_1 distance between S and T. If $|x-y|=1$, we call x and y **nearest neighbors** We will also sometimes need the L_∞ norm given by $\|(x_1,x_2, \ldots ,x_d)\| = \max_i |x_i|$ Everything we will define in this paper will implicitly depend on the dimension d However, we will not mention this d, and all our results will be valid for any $d \geqq 2$. Given a set $R \subseteq \mathbf{Z}^d$, we let $\Delta^v(R) = \{x \notin R : |x-y| = 1 \text{ for some } y \in R\}$ and call this the **vertex boundary of** R We also let $\Delta^e(R) = \{\{x,y\} : x \in R, y \notin R, |x-y| = 1\}$ and call this the **edge boundary of** R

We now introduce probability into the setup For $n = 1,2, \ldots$, let $p_n \in [0,1)$ be such that

$$\sum_{0 \neq x \in \mathbf{Z}^d} p_{|x|} < \infty$$

We picture edges or bonds $\{x,y\}$ between all pairs x and y, $x \neq y$, and declare such an edge to be **open** with probability p_n if $|x-y| = n$ independently of all

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