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Quaternionic Monopoles

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Abstract: We present the simplest non-abelian version of Seiberg–Witten theory: Quaternionic monopoles. These monopoles are associated with $Spin^h(4)$-structures on 4-manifolds and form finite-dimensional moduli spaces. On a Kähler surface the quaternionic monopole equations decouple and lead to the projective vortex equation for holomorphic pairs. This vortex equation comes from a moment map and gives rise to a new complex-geometric stability concept. The moduli spaces of quaternionic monopoles on Kähler surfaces have two closed subspaces, both naturally isomorphic with moduli spaces of canonically stable holomorphic pairs. These components intersect along a Donaldson instanton space and can be compactified with Seiberg–Witten moduli spaces. This should provide a link between the two corresponding theories.

0. Introduction

Recently, Seiberg and Witten [W] introduced new 4-manifold invariants, essentially by counting solutions of the monopole equations. The new invariants have already found nice applications, like e.g. in the proof of the Thom conjecture [KM] or in a short proof of the Van de Ven conjecture [OT2]. In this paper we introduce and study the simplest and the most natural non-abelian version of the Seiberg–Witten monopoles, the quaternionic monopoles.

Let (X, g) be an oriented Riemannian manifold of dimension 4. The structure group $SO(4)$ has as natural extension the quaternionic spinor group $Spin^h(4) = Spin(4) \times_{\mathbb{Z}_2} Sp(1)$:

$$1 \to Sp(1) \to Spin^h(4) \to SO(4) \to 1 .$$

The projection onto the second factor $Sp(1) = SU(2)$ induces a "determinant map" $\delta : Spin^h(4) \to PU(2)$.

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