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Long Time Behavior for the Focusing Nonlinear Schroedinger Equation with Real Spectral Singularities

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Abstract: We consider the effect of real spectral singularities on the long time behavior of the solutions of the focusing Nonlinear Schroedinger equation We find that for each spectral singularity $\lambda' \in \mathbb{R}$, such an effect is limited to the region of the (x,t)-plane in which λ' is close to the point of stationary phase $\lambda_0 = \frac{-x}{4t}$ (the phase here being defined in a standard way by, say, the evolution of the Jost functions) In that region, the solution performs decaying oscillations of the same form as in the other regions, but with different parameters The order of decay is $O((\frac{\log t}{t})^{1/2})$

We prove our result by using the Riemann–Hilbert factorization formulation of the inverse scattering problem We recover our asymptotics by transforming our problem to one which is equivalent for large time, and which can be interpreted as the one corresponding to the genus 0 algebro-geometric solution of the equation

1. Introduction

We consider the nonlinear Schroedinger equation (focusing case)

$$iq_t + q_{xx} + 2q|q|^2 = 0 \tag{1.1}$$

under initial data

$$q(x,0) = q_0(x)\,, \tag{1.2}$$

belonging in the Schwartz class

As is well known (see [NMPZ], [FT]), the problem (1.1)–(1.2) can be integrated through the method of inverse scattering. We will present here some of the results we will need without proof

The associated linear system is

$$\psi_x = \begin{pmatrix} i\lambda & iq(x) \\ i\bar{q}(x) & -i\lambda \end{pmatrix} \psi\,, \tag{1.3}$$