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W-Geometry of the Toda Systems Associated with Non-Exceptional Simple Lie Algebras

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Abstract: The present paper describes the W-geometry of the Abelian finite non-periodic (conformal) Toda systems associated with the B, C and D series of the simple Lie algebras endowed with the canonical gradation. The principal tool here is a generalization of the classical Plücker embedding of the A-case to the flag manifolds associated with the fundamental representations of B_n, C_n and D_n, and a direct proof that the corresponding Kähler potentials satisfy the system of two-dimensional finite non-periodic (conformal) Toda equations It is shown that the W-geometry of the type mentioned above coincide with the differential geometry of special holomorphic (W) surfaces in target spaces which are submanifolds (quadrics) of CP^N with appropriate choices of N In addition, these W-surfaces are defined to satisfy quadratic holomorphic differential conditions that ensure consistency of the generalized Plücker embedding These conditions are automatically fulfilled when Toda equations hold

1. Introduction

A notion of W_φ-geometry of $\mathbf{CP}^N$-target manifolds associated with integrable systems, recently invented in [1] for the case of A_n-Abelian Toda system (see also [2]) seems to be a very important tool for solvable field theories as geometrical structures behind W-algebras, as well as for algebraic and differential geometries themselves In particular, such a geometrical picture should be rather essential in the gauge fields formulation of various models of the two-dimensional gravity, as well as their generalizations for higher dimensions On the same footing as W-algebras, being the algebras of the characteristic integrals – conserved currents – for the corresponding nonlinear systems, guarantee, under appropriate conditions, the integrability property for these systems and give their classification, a description of their W-geometry is equivalent, in a sense, to a classification scheme of

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