

Global Solutions to the Compressible Euler Equations with Geometrical Structure

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Abstract: We prove the existence of global solutions to the Euler equations of compressible isentropic gas dynamics with geometrical structure, including transonic nozzle flow and spherically symmetric flow. Due to the presence of the geometrical source terms, the existence results themselves are new, especially as they pertain to radial flow in an unbounded region, $|\vec{x}| \geq 1$, and to transonic nozzle flow. Arbitrary data with L^∞ bounds are allowed in these results. A shock capturing numerical scheme is introduced to compute such flows and to construct approximate solutions. The convergence and consistency of the approximate solutions generated from this scheme to the global solutions are proved with the aid of a compensated compactness framework.

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1. Introduction

We develop new mathematical existence theory and numerical schemes for global discontinuous solutions to the Euler equations of compressible isentropic gas