On Tangential Star Products
for the Coadjoint Poisson Structure*

M. Cahen[1], **S. Gutt**[1,2], **J. Rawnsley**[3]

[1] Département de Mathématiques, Université Libre de Bruxelles, Campus Plaine CP 218, 1050 Bruxelles,
Belgium Email: mcahen@ulb ac be, sgutt@ulb ac be
[2] Département de Mathématiques, Université de Metz, Ile du Saulcy, F-57045 Metz cedex, France
Email: gutt@poncelet univ-metz fr
[3] Mathematics Institute, University of Warwick, Coventry CV4 7AL, UK
Email: j.rawnsley@warwick ac uk

Abstract: We derive necessary conditions on a Lie algebra from the existence of
a star product on a neighbourhood of the origin in the dual of the Lie algebra for
the coadjoint Poisson structure which is both differential and tangential to all the
coadjoint orbits. In particular we show that when the Lie algebra is semisimple there
are no differential and tangential star products on any neighbourhood of the origin in
the dual of its Lie algebra.

1. Introduction

The notion of star-product on a Poisson manifold, introduced in [1, 2], has been
extensively studied in the literature. The existence of star-products has been shown
in complete generality in the case of symplectic manifolds using several different
approaches. See [5, 6, 11] and the references cited therein. Recently some progress
has been made for Poisson manifolds in various special cases [6, 10, 12].

Any Poisson manifold is foliated by symplectic manifolds [14] called symplectic
leaves and it is natural to ask if star-products can be constructed on Poisson manifolds
by gluing together star-products on the symplectic leaves. Such a star-product we call
tangential. In the case of regular Poisson manifolds where the leaves have constant
dimension the symplectic methods extend to yield tangential star-products.

The simplest Poisson structures are those where the Poisson tensor is linear in
coordinates and these are obtained as the dual of a Lie algebra (the structure constants
are obtained from the Poisson brackets of linear functions). These Poisson tensors are
not regular as they must vanish at the origin. The dual of the Lie algebra of $SU(2)$ has
symplectic leaves which are round 2-spheres centered at the origin and so is regular
outside the origin. We initially tried to see if we could choose Fedosov style star-
products on the leaves in a way which would extend over the origin. To be tangential

---

Correspondence to: J. Rawnsley