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From Dynkin Diagram Symmetries to Fixed Point Structures

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Abstract: Any automorphism of the Dynkin diagram of a symmetrizable Kac-Moody algebra g induces an automorphism of g and a mapping τ_ω between highest weight modules of g. For a large class of such Dynkin diagram automorphisms, we can describe various aspects of these maps in terms of another Kac-Moody algebra, the "orbit Lie algebra" $\breve{\mathsf{g}}$. In particular, the generating function for the trace of τ_ω over weight spaces, which we call the "twining character" of g (with respect to the automorphism), is equal to a character of $\breve{\mathsf{g}}$. The orbit Lie algebras of untwisted affine Lie algebras turn out to be closely related to the fixed point theories that have been introduced in conformal field theory. Orbit Lie algebras and twining characters constitute a crucial step towards solving the fixed point resolution problem in conformal field theory.

1. Introduction

In this paper we associate algebraic structures to automorphisms of Dynkin diagrams and study some of their interrelations. The class of Dynkin diagrams we consider are those of symmetrizable Kac-Moody algebras [1]. These are those Lie algebras which possess both a Cartan matrix and a Killing form, which includes in particular the simple, affine, and hyperbolic Kac-Moody algebras.

An automorphism of a Dynkin diagram is a permutation of its nodes which leaves the diagram invariant. Any such map divides the set of nodes of the diagram into invariant subsets, called the orbits of the automorphism. We focus our attention on two main types of orbits, namely those where each of the nodes on an orbit is either connected by a single link to precisely one node on the same orbit or not linked to any other node on the same orbit. If all orbits of a given Dynkin diagram automorphism are of one of these two types, we say that the automorphism satisfies the *linking condition.* Except for the order N automorphisms of the affine Lie algebras $A^{(1)}_{N-1}$, all diagram automorphisms of simple and affine Lie algebras belong to this class.

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