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Yang–Mills Fields on Cylindrical Manifolds and Holomorphic Bundles II

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Abstract: We give complex holomorphic descriptions of Yang–Mills instantons on tubular four manifolds with nontrivial circle bundles over Riemann surfaces as section.

0. Introduction

Let Y be a nontrivial circle bundle. By the discussion in [8], we know that instantons on $Y \times R$ can be divided into three classes, namely those with flat limits without holonomy along the fibre circle of Y, those with flat limits with holonomy along the fibre circle and those with mixed limits. In [8], we give complex holomorphic descriptions of instantons on $Y \times R$ whose flat limits have trivial holonomy along the fibre circle. In this sequel, we give a complex holomorphic description of instantons whose flat limits have nontrivial holonomy along the fibre circle. The holomorphic data used to describe these instantons is basically different from that in [8], due to the holonomies of the flat limits along the fibre circle of Y. Nevertheless the method used to establish these results is similar to the one used in [8].

We assume the reader is familiar with [8] and shall make constant references to [8], and we shall continue to use the notation introduced in [8].

1. Some Definitions and Statements of the Main Results

Let Y be a circle bundle with non-trivial Chern class over some Riemann surface Σ. Let L and S be the associated line bundle and ruled surface, and also let Σ_0 and Σ_∞ be the two divisors in S as before. By Lemma 3.1 of [8], there is a metric g on Y and a holomorphic structure on $Y \times R$ such that the tube metric $g + dt \otimes dt$ on $Y \times R$ is a Hermitian metric and is conformal to a Kaehler metric. Moreover $Y \times R$ as a complex manifold can be compactified to a ruled surface.

For our main results, first we need to look at the behaviour of this Hermitian tube metric and the Kaehler metric under certain natural maps between tubes $Y \times R$ for different circle bundles Y.