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Integral Representations of the Macdonald Symmetric Polynomials

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Abstract: Multiple-integral representations of the (skew-)Macdonald symmetric polynomials are obtained. Some bosonization schemes for the integral representations are also constructed.

1. Introduction

The Calogero–Sutherland model [1] and its various generalizations [2, 3] have been extensively studied and these $1/r^2$ type models are known to describe systems with the generalized exclusion principle in $1+1$ dimension [4]. The Calogero–Sutherland model describes a system of non-relativistic particles on a circle under the inverse square potential. Its Hamiltonian and momentum are

$$H_{CS} = \sum_{j=1}^{N_0} \frac{1}{2}\left(\frac{1}{i}\frac{\partial}{\partial q_j}\right)^2 + \left(\frac{\pi}{L}\right)^2 \sum_{\substack{i,j=1\\ i<j}}^{N_0} \frac{\beta(\beta-1)}{\sin^2 \frac{\pi}{L}(q_i-q_j)}, \qquad P_{CS} = \sum_{j=1}^{N_0} \frac{1}{i}\frac{\partial}{\partial q_j}, \quad (1.1)$$

where β is a coupling constant. This Calogero–Sutherland model is related to many branches of low-dimensional physics and mathematics: quantum Hall effect [5], 2D Yang–Mills theory [6, 7], matrix model [8, 9], Yangian symmetry [10, 11], Virasoro symmetry [21–23], $W_{1+\infty}$ symmetry [12], Laplace–Beltrami operator [13, 14], etc. One of the recent remarkable developments was the evaluation of some dynamical correlation functions [9, 15–18]. In these calculations the Jack symmetric polynomials [19, 20] play a central role, because they describe the excited states of the Calogero–Sutherland model. In the previous works [21–24], the free field realization of the wave functions, in other words, the integral representations of the Jack symmetric polynomials is discussed.

Several years ago, Ruijsenaars constructed a relativistic (or lattice regularized) version of the Calogero system [25]. That model is integrable, since it has mutually

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