Commun. Math. Phys. 179, 623–646 (1996)

Communications in Mathematical Physics

The $1/N$-Expansion as a Perturbation about the Mean Field Theory: A One-Dimensional Fermi Model

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Received: 5 April 1995/Accepted: 15 January 1996

Abstract: We examine a family of probability measures on $\mathbf{R}^L$ with real parameter $\zeta > 0$ and integer parameters $N, L > 0$. Each such measure is equivalent to the lattice version of a one-dimensional discrete chiral-invariant fermionic quantum field theory with quartic interaction, with N the number of flavours. After applying the Matthews–Salam formula, the model becomes a statistical mechanical model of a chain of continuous Gaussian spins, coupled with a certain non-standard weight function. Finally, the model can also be considered as a probability measure on the set of tridiagonal matrices with fixed off-diagonal and random diagonal entries.

Our analysis shows how to develop an asymptotic expansion in $1/N$, valid for all L and ζ, for the fundamental expectation values. From this it follows that the two point fermion correlation function decays with a mass which agrees to the leading order in $1/N$ with the mean field value calculated by the argument of Gross–Neveu. The analytical technique we develop in essence combines a transfer matrix method with the Laplace method (steepest descent) for asymptotics of integrals.

1. Introduction

It was argued some years ago [NJ, GN] that a chiral invariant fermion theory with quartic interaction will acquire a mass dynamically by spontaneous symmetry breaking. The effective potential shows degenerate minima in the one-loop approximation, leading to a ground state with non-vanishing expectation of $\bar{\psi}\psi$.

Rigorous control of multi-phase theories was developed by Glimm–Jaffe–Spencer [GJS] who devised an expansion about the mean field theory by combining a cluster expansion with an expansion in phase boundaries.

The present paper is part of a program which aims at understanding the mechanism by which mass is generated dynamically in models with four-fermion interactions. We here consider a one-dimensional discrete chiral lattice model and show

★ Partially supported by FAPESP and CNPq.
★★ Partially supported by CNPq.
★★★ Supported by the Natural Sciences and Engineering Research Council of Canada.