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Micro-Local Approach to the Hadamard Condition in Quantum Field Theory on Curved Space-Time

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To my parents

Abstract: For the two-point distribution of a quasi-free Klein-Gordon neutral scalar quantum field on an arbitrary four dimensional globally hyperbolic curved space-time we prove the equivalence of (1) the global Hadamard condition, (2) the property that the Feynman propagator is a distinguished parametrix in the sense of Duistermaat and Hörmander, and (3) a new property referred to as the wave front set spectral condition (WFSSC), because it is reminiscent of the spectral condition in axiomatic quantum field theory on Minkowski space. Results in micro-local analysis such as the propagation of singularities theorem and the uniqueness up to C^∞ of distinguished parametrices are employed in the proof. We include a review of Kay and Wald's rigorous definition of the global Hadamard condition and the theory of distinguished parametrices, specializing to the case of the Klein-Gordon operator on a globally hyperbolic space-time. As an alternative to a recent computation of the wave front set of a globally Hadamard two-point distribution on a globally hyperbolic curved space-time, given elsewhere by Köhler (to correct an incomplete computation in [32]), we present a version of this computation that does not use a deformation argument such as that used in Fulling, Narcowich and Wald and is independent of the Cauchy evolution argument of Fulling, Sweeny and Wald (both of which are relied upon in Köhler's proof). This leads to a simple micro-local proof of the preservation of Hadamard form under Cauchy evolution (first shown by Fulling, Sweeny and Wald) relying only on the propagation of singularities theorem. In another paper [33], the equivalence theorem is used to prove a conjecture by Kay that a locally Hadamard quasi-free Klein-Gordon state on any globally hyperbolic curved space-time must be globally Hadamard.

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