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The Construction of the d + 1-Dimensional Gaussian Droplet

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Dedicated to the memory of Roland Dobrushin, who passed away on 13 November 1995

Abstract: The aim of this note is to study the asymptotic behavior of a gaussian random field, under the condition that the variables are positive and the total volume under the variables converges to some fixed number $v > 0$. In the context of Statistical Mechanics, this corresponds to the problem of constructing a droplet on a hard wall with a given volume. We show that, properly rescaled, the profile of a gaussian configuration converges to a smooth hypersurface, which solves a quadratic variational problem. Our main tool is a scaling dependent large deviation principle for random hypersurfaces.

1. Introduction

What is the most probable shape for a random interface on a wall under the constraint that this interface stays above the wall, is pinned down at its boundary, and moreover that the volume between the wall and the interface is fixed? This question about the shape of a droplet has been treated in one dimension when the random interface (i.e., here a random curve) is a classical random walk, see [3,4]. Dynamics of such droplets are also of interest and are studied in [1], see also [9] for questions dealing with the fluctuations around this most probable shape.

In dimension larger than one, the problem is much harder; we treat here a simple model: the case of a gaussian interface. We show that, under appropriate scaling, a large deviation principle holds which enables us to find this limiting shape as the solution of an elliptic PDE. One of the major difficulties is to control the positivity condition at the boundary of the droplet. We present a scaling-dependent result which relies essentially upon the "entropic repulsion" phenomenon as exhibited in [6,7].

More precisely, let $\Lambda = (0,1)^d$ be the unit cube in $\mathbb{R}^d$, $V_N = N\Lambda \cap \mathbb{Z}^d$, $d \geqq 2$, be the (discrete) box of side $(N-1)$ and set $\Omega_N = \mathbb{R}^{V_N}$. Our a priori distribution is the centered gaussian field P_N^0 on Ω_N, with density with respect to the Lebesgue