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Homotopy Classification of Minimizers of the Ginzburg–Landau Energy and the Existence of Permanent Currents

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Abstract: Our objective is to explain the phenomenon of permanent currents within the context of the Ginzburg–Landau model for superconductors. Using variational techniques we make a connection between the formation of permanent currents and the topology of the superconducting sample.

1. Introduction

Superconductors are materials whose electrical resistivity is effectively zero. They are also known for their peculiar magnetic properties. For example, the magnetic fluxoids, defined precisely in (3.15) below, can only have discrete values. Another interesting property of superconductors is the existence of permanent currents. Such currents are created by submitting a superconducting ring to an external magnetic field. The currents are observed to persist even after the applied field is turned off. The main objective of this paper is to explain the phenomenon of permanent currents and their relation to fluxoids. In particular, we consider the connection between the formation of permanent currents and the topology of the superconducting sample.

We shall use the Ginzburg–Landau theory to model the superconductor. For this purpose we denote the superconducting electrons density by $u(x)$, and set A to be the magnetic vector potential. In the absence of an applied magnetic field, the energy is described by the functional (see e.g. [A, DGP])

$$E_\varepsilon(u,A) = \int_\Omega \frac{1}{2}|(\nabla - iA)u|^2 + \varepsilon^{-2}V(u)\,dx + \int_{\mathbf{R}^3} \frac{1}{2}|\nabla \times A|^2\,dx\,, \tag{1.1}$$

where $V(u) = \frac{1}{4}(|u|^2 - 1)^2$, Ω is a bounded domain in $\mathbf{R}^3$, u is a complex-valued function defined on Ω, $A : \mathbf{R}^3 \to \mathbf{R}^3$ and ε^{-1} is the Ginzburg–Landau parameter. Note that (1.1) consists of two terms. The first is the energy associated with the superconducting electrons, which are confined to the domain Ω, occupied by the

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