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Almost Flat Planar Diagrams

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In memoriam Claude Itzykson

Abstract: We continue our study of matrix models of dually weighted graphs. Among the attractive features of these models is the possibility to interpolate between ensembles of regular and random two-dimensional lattices, relevant for the study of the crossover from two-dimensional flat space to two-dimensional quantum gravity. We further develop the formalism of large N character expansions. In particular, a general method for determining the large N limit of a character is derived. This method, aside from being potentially useful for a far greater class of problems, allows us to exactly solve the matrix models of dually weighted graphs, reducing them to a well-posed Riemann–Hilbert problem. The power of the method is illustrated by explicitly solving a new model in which only positive curvature defects are permitted on the surface, an arbitrary amount of negative curvature being introduced at a single insertion.

1. Introduction

Hermitian one matrix models were introduced and for the first time solved in the large N limit in the seminal paper by Brézin, Itzykson, Parisi and Zuber [1]. These models generate ensembles of planar, random graphs whose vertex coordination numbers are controlled by the matrix potential. By varying the potential, different classes of diagrams may be obtained, e.g. random square or random triangular lattices. However, despite this freedom, there is a class of physically important lattices that cannot be generated by simply tuning the potential: *regular*, flat lattices with fixed coordination numbers of both vertices and faces. To attain them it is necessary to study planar graphs having coordination number dependent weights

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