Commun. Math. Phys. 179, 215 – 234 (1996)

Communications in Mathematical Physics

Zeta Function Determinant of the Laplace Operator on the D-Dimensional Ball

M. Bordag[1,★], **B. Geyer**[1,★★], **K. Kirsten**[1,★★★], **E. Elizalde**[2,★★★★]

[1] Universität Leipzig, Institut für Theoretische Physik, Augustusplatz 10, D-04109 Leipzig, Germany

[2] CEAB, CSIC, Camí de Santa Bàrbara, 17300 Blanes, and Departament d'ECM and IFAE, Facultat de Física, Universitat de Barcelona, Av. Diagonal 647, E-08028 Barcelona, Spain

Received: 18 May 1995 / Accepted: 23 October 1995

Abstract: We present a direct approach for the calculation of functional determinants of the Laplace operator on balls. Dirichlet and Robin boundary conditions are considered. Using this approach, formulas for any value of the dimension, D, of the ball, can be obtained quite easily. Explicit results are presented here for dimensions $D = 2$, 3, 4, 5 and 6.

1. Introduction

Motivated by the need to give answers to some fundamental questions in quantum field theory, during the last years there has been (and continues to be) a lot of interest in the problem of calculating the determinant of a differential operator, L (see for example [1, 2]). Often one has to deal in these situations with positive elliptic differential operators acting on sections of a vector bundle over a compact manifold. In such cases L has a discrete spectrum $\lambda_1 \leqq \lambda_2 \leqq \cdots \rightarrow \infty$. The determinant, $\det L = \prod_i \lambda_i$, is generally divergent and one needs to make sense out of it by means of some kind of analytic continuation. A most appropriate way of doing that is by using the zeta function regularization prescription introduced by Ray and Singer [3] (see also [4, 5]). In this procedure $\ln \det L$ is defined by analytically continuing the function $\sum_i \lambda_i^{-s} \ln \lambda_i$ in the exponent s, from the domain of the complex plane where the real part of s is large to the point $s = 0$. Introducing the zeta function associated with the spectrum λ_i of L,

$$\zeta(s) = \sum_i \lambda_i^{-s} ,$$

this is equivalent to defining

$$\ln \det L = -\zeta'(0) .$$

★ E-mail address: bordag@qft.physik.uni-leipzig.d400.de.

★★ E-mail address: geyer@ntz.uni-leipzig.d400.de.

★★★ E-mail address: kirsten@tph100.physik.uni-leipzig.de.

★★★★ E-mail address: eli@zeta.ecm.ub.es.