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Target-Space Duality between Simple Compact Lie Groups and Lie Algebras under the Hamiltonian Formalism: I. Remnants of Duality at the Classical Level

Orlando Alvarez*, **Chien-Hao Liu****

Department of Physics, University of Miami, P.O. Box 248046, Coral Gables, FL 33124, USA

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Abstract: It has been suggested that a possible classical remnant of the phenomenon of target-space duality (T-duality) would be the equivalence of the classical string Hamiltonian systems. Given a simple compact Lie group G with a bi-invariant metric and a generating function Γ suggested in the physics literature, we follow the above line of thought and work out the canonical transformation Φ generated by Γ together with an Ad-invariant metric and a B-field on the associated Lie algebra $\mathfrak{g}$ of G so that G and $\mathfrak{g}$ form a string target-space dual pair at the classical level under the Hamiltonian formalism. In this article, some general features of this Hamiltonian setting are discussed. We study properties of the canonical transformation Φ including a careful analysis of its domain and image. The geometry of the T-dual structure on $\mathfrak{g}$ is lightly touched. We leave the task of tracing back the Hamiltonian formalism at the quantum level to the sequel of this paper.

0. Introduction and Outline

0.1. Introduction. Target space duality (T-duality) is a very surprising phenomenon in string theory[1]. In essence, two target-spaces are dual to each other if both lead to the same string theory. The usual technical definition involves using path-integrals to sum over the space of all smooth maps from surfaces (string world-sheets) to target manifolds [B1, B2, F-J, R-V, G-R1, M-V]. In this aspect, it is a quantum mechanical phenomenon. Nevertheless, it is natural to ask:

"*Q*: *Are there classical aspects of the phenomenon of target space duality*?

As already pointed out in the literature (e.g. [A-AG-B-L, A-AG-L2, C-Z, G-P-R, G-R3, G-R-V]), one possible answer may be the equivalence of the associated string Hamiltonian systems.

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* e-mail: alvarez@phyvax.ir.miami.edu.

** e-mail: chienliu@phyvax.ir.miami.edu.

[1] See the review [G-P-R] for a comprehensive set of references.