

Induced Modules for Vertex Operator Algebras

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Abstract: For a vertex operator algebra V and a vertex operator subalgebra V' which is invariant under an automorphism g of V of finite order, we introduce a g -twisted induction functor from the category of g -twisted V' -modules to the category of g -twisted V -modules. This functor satisfies the Frobenius reciprocity and transitivity. The results are illustrated with V' being the g -invariants in simple V or V' being g -rational.

1. Introduction

A lot of progress on the representation theory for vertex operator algebras has been made in the last few years. For example, the representation theory for the concrete vertex operator algebras, which include the moonshine vertex operator algebra $V^\#$ ([FLM, D3]), the vertex operator algebras based on even positive definite lattices [D1], the vertex operator algebras associated with the integrable representations of affine Lie algebras and Virasoro algebras ([DMZ, DL, FZ, W]), have been studied extensively. There are also abstract approaches such as Zhu's one to one correspondence between the set of inequivalent irreducible modules for a given vertex operator algebra and the set of inequivalent irreducible modules for an associative algebra associated with the vertex operator algebra [Z], and the tensor products of modules ([HL, L]); see also [FHL] for the results concerning intertwining operators and contragredient modules. Many of these results are analogues of the corresponding results in the classical Lie algebra theory.

The purpose of this paper is to give a construction of induced twisted modules for vertex operator algebras and present some initial results. The main idea in constructing the induced module comes from the induction theory for the representations of Lie groups, algebraic groups, quantum groups, Hopf algebras ([V, J, APW, Lin1–Lin2]). In order for the induced module to have the functorial

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