Commun. Math. Phys. 179, 121 – 156 (1996)

Communications in Mathematical Physics

Automorphism Modular Invariants of Current Algebras

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Received: 3 April 1995 / Accepted: 23 October 1995

Abstract: We consider those two-dimensional rational conformal field theories (RCFTs) whose chiral algebras, when maximally extended, are isomorphic to the current algebra formed from some untwisted affine Lie algebra at fixed level. In this case the partition function is specified by an automorphism of the fusion ring and corresponding symmetry of the Kac–Peterson modular matrices. We classify all such partition functions when the underlying finite-dimensional Lie algebra is simple. This gives all possible spectra for this class of RCFTs. While accomplishing this, we also find the primary fields with second smallest quantum dimension.

1. Introduction

In two-dimensional conformal field theory, scale invariance means boundary conditions have an impact on the local physics, even far from a boundary [6]. For example, a conformal field theory must be consistent on the interior of a parallelogram with periodic boundary conditions imposed, i.e. on a torus. In particular, the corresponding partition function should not be sensitive to changes of the modular parameter that keep a torus within the same conformal class. The partition function must be modular invariant.

The local symmetry of the conformal field theory also constrains the partition function. The chiral algebra of currents determines the conformal blocks [2] of the torus partition function. That is, the partition function must be a sesquilinear combination of characters of the chiral algebra. The two constraints together often determine the field content of a given conformal field theory. This analysis of conformal field theories is known as the modular bootstrap.

We apply the modular bootstrap program to conformal field theories whose (maximal) chiral algebras are isomorphic to the current algebra of untwisted affine Lie algebras at fixed levels. We call such algebras *conformal current algebras*, and

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** Supported in part by NSERC.