Conformal Field Theories, Representations and Lattice Constructions

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Abstract: An account is given of the structure and representations of chiral bosonic meromorphic conformal field theories (CFT's), and, in particular, the conditions under which such a CFT may be extended by a representation to form a new theory. This general approach is illustrated by considering the untwisted and Z_2-twisted theories, $\mathcal{H}(\Lambda)$ and $\tilde{\mathcal{H}}(\Lambda)$ respectively, which may be constructed from a suitable even Euclidean lattice Λ. Similarly, one may construct lattices $\Lambda_\mathscr{C}$ and $\tilde{\Lambda}_\mathscr{C}$ by analogous constructions from a doubly-even binary code $\mathscr{C}$. In the case when $\mathscr{C}$ is self-dual, the corresponding lattices are also. Similarly, $\mathcal{H}(\Lambda)$ and $\tilde{\mathcal{H}}(\Lambda)$ are self-dual if and only if Λ is. We show that $\mathcal{H}(\Lambda_\mathscr{C})$ has a natural "triality" structure, which induces an isomorphism $\mathcal{H}(\tilde{\Lambda}_\mathscr{C}) \equiv \tilde{\mathcal{H}}(\Lambda_\mathscr{C})$ and also a triality structure on $\tilde{\mathcal{H}}(\tilde{\Lambda}_\mathscr{C})$. For $\mathscr{C}$ the Golay code, $\tilde{\Lambda}_\mathscr{C}$ is the Leech lattice, and the triality on $\tilde{\mathcal{H}}(\tilde{\Lambda}_\mathscr{C})$ is the symmetry which extends the natural action of (an extension of) Conway's group on this theory to the Monster, so setting triality and Frenkel, Lepowsky and Meurman's construction of the natural Monster module in a more general context. The results also serve to shed some light on the classification of self-dual CFT's. We find that of the 48 theories $\mathcal{H}(\Lambda)$ and $\tilde{\mathcal{H}}(\Lambda)$ with central charge 24 that there are 39 distinct ones, and further that all 9 coincidences are accounted for by the isomorphism detailed above, induced by the existence of a doubly-even self-dual binary code.

1. Introduction

In this paper we shall provide the details omitted from the summary of our results given in [1].

The principal result of the paper will be to show how a study of binary linear codes leads to an understanding of some of the symmetries of conformal field theories (CFT's). We shall restrict ourselves of self-dual chiral bosonic theories, which are regarded as trivial by approaches to the CFT classification problem which rely upon a study of the fusion rules for the representations of some chiral algebras [2]. (For general reviews of CFT see [3,4].) Hence, a complete understanding of