

# Factorisations for Partition Functions of Random Hermitian Matrix Models

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**Abstract:** The partition function  $Z_N$ , for Hermitian-complex matrix models can be expressed as an explicit integral over  $\mathbb{R}^N$ , where  $N$  is a positive integer. Such an integral also occurs in connexion with random surfaces and models of two dimensional quantum gravity. We show that  $Z_N$  can be expressed as the product of two partition functions, evaluated at translated arguments, for another model, giving an explicit connexion between the two models. We also give an alternative computation of the partition function for the  $\phi^4$ -model. The approach is an algebraic one and holds for the functions regarded as formal power series in the appropriate ring.

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