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Time Decay of Solutions of Coupled Maxwell-Klein-Gordon Equations

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Abstract: We obtain the optimal time decay of the solutions of the coupled Maxwell-Klein-Gordon equations in four dimensional spacetime, provided the initial data are what we define as Coulomb. In other words, the initial data are such that the Klein-Gordon field is smooth and compactly supported and the Maxwell field is electrostatic outside this support. The problem involves charge, therefore, the initial data do not satisfy either fast decay or any smallness condition. In spite of that, we are able to obtain our result using the inversion map of the lightcone of a carefully selected origin. We thus, avoided the blow-up that takes place when using the usual conformal transformation to the Einstein spacetime.

1. Introduction

The presence of charge imposes precise conditions on the decay of the initial electromagnetic field preventing it from being placed in a weighted Sobolev space. Therefore, the usual techniques of conformal transformation or smallness of initial data to prove decay do not work.

The condition that initially the electromagnetic field outside the (compact) support of the Klein-Gordon field is static, permits us to use the inversion map at the lightcone of a carefully selected origin. The conformal covariance of the equations involved and the global existence theorem (Theorem 1) allow us to obtain the optimal time decay for the solutions. This method can not be used with a non-Abelian gauge group because the commutation relation would prevent the fields from being static (the commutators would behave like a source with non-compact support).

In the usual conformal compactification of Penrose-Christodoulou, the generator of the transformation is a conformal Killing field of the Minkowski metric in the whole of spacetime. In contrast, the generator of the inversion is a conformal Killing field in the interior of the lightcone at the point on which the inversion is made. Thus, while with the Penrose-Christodoulou method we have to take into account

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