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Deformations of Compact Quantum Groups via Rieffel's Quantization

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Abstract: It is shown that compact quantum groups containing torus subgroups can be deformed into new compact quantum groups under Rieffel's quantization. This is applied to showing that the two classes of compact quantum groups K_q^u and K_q studied by Levendorkii and Soibelman are strict deformation quantization of each other, and that the quantum groups $A_u(m)$ have many deformations.

1. Introduction

This paper answers in the affirmative the following two questions of Rieffel's: (1) Are Drinfeld's algebraic twistings K_q^u of the quantum groups K_q, as studied in [20, 12, 13], strict deformation quantizations of K_q? (2) Can the quantum groups $A_u(m)$ constructed in [25, 26] be deformed? The key to answering these questions is a result, in the spirit of [17], on deformations of arbitrary compact quantum groups (instead of only compact groups as treated there). We believe this result is of interest in its own right.

We now describe the results of this paper in more detail. Let A be a Woronowicz Hopf C^*-algebra in the sense of [30, 2, 25, 26], whose coproduct is denoted by Φ. We will also call it a compact quantum group, referring to its dual object (cf. [26]). Suppose that the quantum group A has an abelian Lie subgroup T. This means that there is a surjective C^*-algebra homomorphism π from A to $C(T)$ preserving the coproducts (see [25, 26]). For any element h in T, denote by E_h the corresponding evaluation functional on $C(T)$. Assume that η is a continuous homomorphism from a vector space Lie group $\mathbb{R}^n$ to T, where n is allowed to be different from the dimension of T. Define an action α of $\mathbb{R}^d := \mathbb{R}^n \times \mathbb{R}^n$ on the C^*-algebra A as follows:

$$\alpha_{(s,u)} = \lambda_{\eta(s)}\rho_{\eta(u)}.$$

In the above,

$$\lambda_{\eta(s)} = (E_{\eta(-s)}\pi \otimes id)\Phi, \quad \rho_{\eta(u)} = (id \otimes E_{\eta(u)}\pi)\Phi,$$

where id is the identity map on A. For any skew-symmetric operator S on $\mathbb{R}^n$, one may apply Rieffel's quantization procedure [16] for the action α above to obtain a