Commun. Math. Phys. 178, 703–732 (1996)

Communications in
Mathematical
Physics

High Temperature Expansions and Dynamical Systems

J. Bricmont[1,★], **A. Kupiainen**[2,★★]
[1] UCL, Physique Théorique, B-1348, Louvain-la-Neuve, Belgium. bricmont@fyma.ucl.ac.be
[2] Helsinki University, Department of Mathematics, Helsinki 00014, Finland. ajkupiai@cc.helsinki.fi

Received: 28 April 1995/Accepted: 10 October 1995

Abstract: We develop a resummed high-temperature expansion for lattice spin systems with long range interactions, in models where the free energy is not, in general, analytic. We establish uniqueness of the Gibbs state and exponential decay of the correlation functions. Then, we apply this expansion to the Perron–Frobenius operator of weakly coupled map lattices.

1. Introduction

The theory of Gibbs states was originally developed for the mathematical analysis of equilibrium statistical mechanics. An interesting application of the theory was found by Sinai, Ruelle and Bowen in the 70's [42, 43, 39, 1] who applied it to the ergodic theory of uniformly hyperbolic dynamical systems. While this so-called thermodynamic formalism has been very successful in ergodic theory, the Gibbs states that describe the statistics of such dynamical systems are quite simple from the point of view of statistical mechanics: they describe one dimensional spin systems with spins taking values in a finite set and interacting with exponentially decaying potentials. In particular, phase transitions, i.e. the coexistence of several Gibbs states for the same interaction, which are of major interest in statistical mechanics, are absent in such systems.

More recently, it has been realized that certain infinite dimensional dynamical systems possess attracting sets that are *extensive* in a suitable volume. This is believed to be the case for many classes of nonlinear parabolic partial differential equations on some spatial domain: the dimension of the attracting set (or a bound for it) increases to infinity as the domain becomes unbounded [44]. Discrete time dynamical systems, such as coupled maps, were introduced to model these phenomena [29]. Bunimovich and Sinai [5] showed that these systems give rise to a thermodynamic formalism for spin systems on a lattice of more than one dimension. Because of this last feature, the possibility of phase transitions is at least open.

★ Supported by EC grants SC1-CT91-0695 and CHRX-CT93-0411.
★★ Supported by NSF grant DMS-9205296 and EC grant CHRX-CT93-0411.