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Quantum $\mathcal{W}$-Algebras and Elliptic Algebras

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Abstract: We define a quantum $\mathcal{W}$-algebra associated to $\mathfrak{sl}_N$ as an associative algebra depending on two parameters. For special values of the parameters, this algebra becomes the ordinary $\mathcal{W}$-algebra of $\mathfrak{sl}_N$, or the q-deformed classical $\mathcal{W}$-algebra of $\mathfrak{sl}_N$. We construct free field realizations of the quantum $\mathcal{W}$-algebras and the screening currents. We also point out some interesting elliptic structures arising in these algebras. In particular, we show that the screening currents satisfy elliptic analogues of the Drinfeld relations in $U_q(\widehat{\mathfrak{n}})$.

1. Introduction

1.1. In [1] N. Reshetikhin and the second author introduced new Poisson algebras $\mathcal{W}_q(\mathfrak{g})$, which are q-deformations of the classical $\mathcal{W}$-algebras. The Poisson algebra $\mathcal{W}_q(\mathfrak{g})$ is by definition the center of the quantized universal enveloping algebra $U_q(\widehat{\mathfrak{g}}^L)$ at the critical level, where $\mathfrak{g}^L$ is the Langlands dual Lie algebra to $\mathfrak{g}$. It was shown in [1] that the Wakimoto realization of $U_q(\widehat{\mathfrak{sl}}_N)$ constructed in [2] provides a homomorphism from the center of $U_q(\widehat{\mathfrak{sl}}_N)$ to a Heisenberg–Poisson algebra $\mathcal{H}_q(\mathfrak{sl}_N)$. This homomorphism can be viewed as a free field realization of $\mathcal{W}_q(\mathfrak{sl}_N)$. When $q = 1$, it becomes the well-known Miura transformation [3]. In [1] explicit formulas for this free field realization were given. The structure of these formulas is the same as that of the formulas for the spectra of transfer-matrices in integrable quantum spin chains obtained by the Bethe ansatz method [4]. This is not surprising given that these spectra can actually be computed using the center at the critical level and the Wakimoto realization. For the Gaudin models, which correspond to the $q = 1$ case, this was explained in detail in [5].

1.2. The Poisson algebra $\mathcal{W}_q(\mathfrak{sl}_2)$ is a q-deformation of the classical Virasoro algebra. It has generators t_n, $n \in \mathbb{Z}$. The relations in $\mathcal{W}_q(\mathfrak{sl}_2)$ were computed in [1] using the q-deformed Miura transformation, which is a homomorphism from $\mathcal{W}_q(\mathfrak{sl}_2)$ to

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