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On a Model for Quantum Friction III. Ergodic Properties of the Spin – Boson System

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Abstract: We investigate the dynamics of a 2-level atom (or spin $\frac{1}{2}$) coupled to a mass-less bosonic field at positive temperature. We prove that, at small coupling, the combined quantum system approaches thermal equilibrium. Moreover we establish that this approach is exponentially fast in time. We first reduce the question to a spectral problem for the Liouvillean, a self-adjoint operator naturally associated with the system. To compute this operator, we invoke Tomita-Takesaki theory. Once this is done we use complex deformation techniques to study its spectrum. The corresponding zero temperature model is also reviewed and compared. From a more philosophical point of view our results show that, contrary to the conventional wisdom, quantum dynamics can be simpler at positive than at zero temperature.

1. Introduction

In this paper we consider the dissipative dynamics of a quantum mechanical 2-level system – the spin – characterized by its two eigenstates of energy $e_\pm = \pm 1$. More specifically we investigate the long time behavior of the dynamics of a spin $\frac{1}{2}$ allowed to interact with a large reservoir. The reservoir is an infinitely extended gas of free, mass-less bosons at positive temperature without Bose-Einstein condensate. We prove that, for sufficiently small coupling, the interacting *spin-boson* system has strong ergodic properties. In particular it approaches thermal equilibrium exponentially fast. Moreover, the equilibrium state is the unique KMS state of the joint system at the temperature of the heat bath.

The spin-boson system is a simple, yet physically acceptable model for a variety of phenomena related to dissipative quantum tunneling. The literature on the subject is enormous. Let us only mention the review article [LCD] as an excellent introduction to the physical aspects of the model. Also [A1, A2, AM, FNV1, FNV2, FNV3, D1, D2, HS1, HS2, MA, PU, SD, SDLL, RO1, RO2] is a non-exhaustive list of related mathematical investigations.