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Deformation of a Kac Algebra by an Abelian Subgroup

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Dedicated to the memory of George Kac (Георгий Исаакович Кац)

Abstract: Some examples of quantum groups in literature arise as deformations of a locally compact group by a "dual" 2-cocycle. We make this construction in the framework of Kac algebras; we show that these deformations are still Kac algebras; using this construction, we give new quantizations of the Heisenberg group. From this point of view, we analyse the dimension 8 non-trivial example of Kac and Paljutkin, and give a new example of non-trivial dimension 12 semi-simple $*$-Hopf algebras (a dimension 12 Kac algebra).

1. Introduction

1.1. The first attempts to define group-like structures (what is now called "quantum groups") with the help of operator algebras were made to clarify the duality of locally compact (non-abelian) groups, this group-like structure then to be put on the "dual" of the locally compact group. At the algebraic level, the notion of Hopf algebras is commonly used to deal with discrete groups and their dual objects at the same time, and therefore, to reach the same aim for general locally compact groups, because of the analysis of infinite dimensional unitary representations, it was more or less natural to mix up operator algebras and the algebraic framework of Hopf algebras. This was made in the 60's by G.I. Kac, who constructed the "ring-groups," a category which contained both unimodular groups and their duals; the non-unimodular case, i.e. to construct a wider category which contains both locally compact groups and their duals, was done in the 70's, after the works of M. Takesaki, independently by G.I. Kac and the second author, and by J.-M. Schwartz and the first author, who named "Kac algebras" this wider category, in order to emphasize Kac's 1961 basic work. On that theory, we refer to [ES2].

1.2. Unfortunately, this theory suffered for a long time from a serious lack of non-trivial examples (see [KP1] and [KP2] for the first non-trivial examples); on

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