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Four Dimensional Realization of Two Dimensional Current Groups

Igor B. Frenkel, Boris A. Khesin

Department of Mathematics, Yale University, New Haven, CT 06520, USA

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Abstract: The quotient realization of the central extensions of the current groups over Riemann surfaces is achieved by means of the Leray residue theory. This approach replaces de Rham cohomology in the classical WZNW construction for affine Lie groups.

Introduction

In the past decade the theory of affine Lie algebras and of the corresponding groups has become an established field in mathematics and theoretical physics with diverse connections to other areas in both disciplines. The geometric aspect of the theory was significantly advanced with an explicit realization of the affine Lie groups as quotients of central extensions of the current groups over a two-dimensional disk [W1, PS, M]. This construction is achieved by a striking combination of the basic facts from the theory of simple Lie groups and of de Rham cohomology. The realization of affine Lie groups is also directly related to the geometric approach to representation theory and a class of two-dimensional conformal field theories known as the WZNW model.

There were numerous attempts to generalize affine Lie algebras and groups to higher dimensions. However, corresponding generalizations of representation theory as well as of conformal field theory always encountered substantial difficulties. In [EF] there was introduced a class of central extensions of two dimensional current algebras and groups on a Riemann surface Σ (endowed with a complex structure) with values in a complex simple Lie group G that can be viewed as a natural generalization of the one dimensional counterpart. In particular, it was shown in [EF, EK] that the orbits in the coadjoint representation for these groups have a structure similar to those for loop groups. This fact strongly indicates that there exists a rich representation theory that generalizes the classical case of affine Lie groups. It was also shown that the central extension $\hat{G}^{\Sigma}$ of the current group $G^{\Sigma} = C^{\infty}(\Sigma, G)$ topologically is a nontrivial fibre bundle with a fibre isomorphic to the Jacobian variety of Σ [EF].