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The Adiabatic Approximation for Quantum Spin Systems with a Spectral Gap

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Summary. We estimate the accuracy of the adiabatic approximation in predicting the time evolution of local observables for an XY quantum magnet with a slowly variable external magnetic field. The system evolves according to the natural Hamiltonian dynamics and the spectral gap produced by the magnetic field is assumed to be large with respect to the term inducing quantum fluctutions. The proof is based on a finite order truncation of a time dependent cluster expansion in inverse powers of the time scale τ. In the analytic case, we show that the accuracy of this truncated expansion is of order $O(\mathrm{e}^{-\alpha \mathrm{e} \cdot \tau^{\frac{1}{\alpha}}})$ for any $\alpha > 1$. If the time dependent perturbation is suddenly switched on at time zero and switched off at time τ, the accuracy of the adiabatic approximation is proven to be of order $O(\tau^{-1})$.

1. Introduction

The adiabatic approximation and linear response theory are basic tools of non-equilibrium statistical mechanics which are useful for setting up the memory function formalism and understanding the hydrodynamic description [F]. In this paper, we study a quantum spin system at zero temperature for which the accuracy of these approximations can be rigorously controlled. The model is described by a time dependent Hamiltonian $\tau\mathbb{H}(t)$, where $t \in [0, 1]$ is a rescaled time coordinate and $\tau \gg 1$ is the time scale. We assume that the time dependency is analytic because the study of this case appears to be more instructive. However, weaker formulations of the two results of this article hold also in case the degree of smoothness is finite. Each operator $\mathbb{H}(t)$ at fixed time t has a gap of order 1 and a ground state with short range correlations which can be computed in perturbation theory. If one chooses as initial condition the instantaneous ground state at time zero, then at time $t = 1$ we prove that the system is in the ground state of $\mathbb{H}(1)$ up to errors that, if measured by computing the expectation of a local observable, are of order τ^{-1}. The subleading corrections can be computed in terms of an asymptotic expansion in inverse powers of τ. The