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Bethe Subalgebras in Twisted Yangians

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Abstract: We study analogues of the Yangian of the Lie algebra $\mathfrak{gl}_N$ for the other classical Lie algebras $\mathfrak{so}_N$ and $\mathfrak{sp}_N$. We call them twisted Yangians. They are coideal subalgebras in the Yangian of $\mathfrak{gl}_N$ and admit homomorphisms onto the universal enveloping algebras $\mathrm{U}(\mathfrak{so}_N)$ and $\mathrm{U}(\mathfrak{sp}_N)$ respectively. In every twisted Yangian we construct a family of maximal commutative subalgebras parametrized by the regular semisimple elements of the corresponding classical Lie algebra. The images in $\mathrm{U}(\mathfrak{so}_N)$ and $\mathrm{U}(\mathfrak{sp}_N)$ of these subalgebras are also maximal commutative.

Introduction

In this article we study the Yangian of the Lie algebra $\mathfrak{gl}_N$ and its analogues for the other classical Lie algebras $\mathfrak{so}_N$ and $\mathfrak{sp}_N$. The Yangian $\mathrm{Y}(\mathfrak{gl}_N)$ is a deformation of the universal enveloping algebra $\mathrm{U}(\mathfrak{gl}_N[t])$ in the class of Hopf algebras [D1]. Moreover, it contains the universal enveloping algebra $\mathrm{U}(\mathfrak{gl}_N)$ as a subalgebra and admits a homomorphism $\pi : \mathrm{Y}(\mathfrak{gl}_N) \to \mathrm{U}(\mathfrak{gl}_N)$ identical on $\mathrm{U}(\mathfrak{gl}_N)$.

Let $\mathfrak{a}_N$ be one of the Lie algebras $\mathfrak{so}_N$ and $\mathfrak{sp}_N$. In [D1] the Yangian $\mathrm{Y}(\mathfrak{a}_N)$ was defined as a deformation of the Hopf algebra $\mathrm{U}(\mathfrak{a}_N[t])$. It contains $\mathrm{U}(\mathfrak{a}_N)$ as a subalgebra but does not admit a homomorphism $\mathrm{Y}(\mathfrak{a}_N) \to \mathrm{U}(\mathfrak{a}_N)$ identical on $\mathrm{U}(\mathfrak{a}_N)$. In the present article we consider another analogue of the Yangian $\mathrm{Y}(\mathfrak{gl}_N)$ for the classical Lie algebra $\mathfrak{a}_N$. It has been introduced in [O2] and called the twisted Yangian; see also [MNO]. The definition in [O2] was motivated by [O1] and [C2, S]. Algebras closely related to this analogue of $\mathrm{Y}(\mathfrak{gl}_N)$ were recently studied in [NS].

Consider $\mathfrak{a}_N$ as a fixed point subalgebra in the Lie algebra $\mathfrak{gl}_N$ with respect to an involutive automorphism σ. The twisted Yangian $\mathrm{Y}(\mathfrak{gl}_N, \sigma)$ is a subalgebra in $\mathrm{Y}(\mathfrak{gl}_N)$. Moreover, it is a left coideal in the Hopf algebra $\mathrm{Y}(\mathfrak{gl}_N)$. It also contains $\mathrm{U}(\mathfrak{a}_N)$ as a subalgebra and does admit a homomorphism $\rho : \mathrm{Y}(\mathfrak{gl}_N, \sigma) \to \mathrm{U}(\mathfrak{a}_N)$ identical on $\mathrm{U}(\mathfrak{a}_N)$; see Sect. 3. The algebra $\mathrm{Y}(\mathfrak{gl}_N, \sigma)$ is a deformation of the universal enveloping algebra for the twisted current Lie algebra

$$\{F(t) \in \mathfrak{gl}_N[t] \mid \sigma(F(t)) = F(-t)\} .$$