Commun. Math. Phys. 178, 425–452 (1996)

Communications in Mathematical Physics


# Exact Operator Solution of the Calogero–Sutherland Model

**Luc Lapointe, Luc Vinet**

Centre de Recherches Mathématiques, Université de Montréal, C.P. 6128, succursale Centre-ville, Montréal, Québec, Canada, H3C 3J7

Received: 17 July 1995/Accepted: 6 September 1995

**Abstract:** The wave functions of the Calogero–Sutherland model are known to be expressible in terms of Jack polynomials. A formula which allows to obtain the wave functions of the excited states by acting with a string of creation operators on the wave function of the ground state is presented and derived. The creation operators that enter in this formula of Rodrigues-type for the Jack polynomials involve Dunkl operators.

## 1. Introduction

Exactly solvable models are of great help in the understanding of quantum many-body physics. The Calogero–Sutherland (CS) [1, 2, 3] model, which describes a system of $N$ particles on a circle interacting pairwise through long range potentials, is generating a lot of attention in this connection, in particular because it provides a fully solvable model in which the ideas of fractional statistics can be tested [4]. There is thus considerable interest in identifying the algebraic structure responsible for the solvability of this model.

The spectrum of the CS Hamiltonian can be interpreted as the energy of a collection of free quasi-particles obeying a generalized exclusion principle. Recent computations [5, 6, 7] of some correlation functions have confirmed this point of view and shown that the exclusion statistics of quasi-particles and quasi-holes is consistent with the anyon statistics of the real particles. The calculation of these quantities proved possible because of the following circumstance: the eigenfunctions of the CS Hamiltonian are given in terms of Jack polynomials [8, 9, 10]. These polynomials form a basis for the ring of symmetric functions and enjoy algebraic properties that allow to carry out analytically the computation of various dynamical functions of the CS model. These polynomials also appear in related areas like the characterization of the Virasoro and $W_N$ algebras singular vectors [11, 12] and in the construction of Yangian modules [13, 14].

We will show in this paper that the wave functions of the CS Hamiltonian and hence the Jack polynomials can be obtained by applying a string of creation