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Algebraic Quantization, Good Operators and Fractional Quantum Numbers

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Abstract: The problems arising when quantizing systems with periodic boundary conditions are analysed, in an algebraic (group-) quantization scheme, and the "failure" of the Ehrenfest theorem is clarified in terms of the already defined notion of *good* (and *bad*) operators. The analysis of "constrained" Heisenberg–Weyl groups according to this quantization scheme reveals the possibility for quantum operators without classical analogue and for new quantum (fractional) numbers extending those allowed for Chern classes in traditional Geometric Quantization. This study is illustrated with the examples of the free particle on the circumference and the charged particle in a homogeneous magnetic field on the torus, both examples featuring "anomalous" operators, non-equivalent quantization and the latter, fractional quantum numbers. These provide the rationale behind flux quantization in superconducting rings and Fractional Quantum Hall Effect, respectively.

1. Introduction

The need for a consistent quantization scheme which is truly suitable for systems wearing a non-trivial topology is increasing daily. Configuration spaces with non-trivial topology appear in as diverse cases as Gauge Theories, Quantum Gravity, and the more palpable ones of the superconducting ring and the Quantum Hall effect, where the measuring tools change the topology of the system in a non-trivial way [I, B-M-S-S, L-W, L-Li].

The most common problem which appears when the configuration-space manifold possesses a non-trivial topology is the failure of the Ehrenfest theorem for certain operators, a problem usually referred to as an anomaly. In the sequel, we shall add the qualifier *topologic* to distinguish these from others directly attached to the Lie algebra of the quantum operators and characterized, roughly speaking, by the appearance of a term in a quantum commutator not present at the classical, Poisson-algebra level. We call them *algebraic* anomalies and refer the reader to [A-N-B-L] for a detailed analysis.

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