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# On the Breakdown of Axisymmetric Smooth Solutions for the 3-D Euler Equations

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**Abstract:** We refine the Beale–Kato–Majda criterion for the breakdown of smooth solutions of the 3-D incompressible Euler equations in the case of axisymmetry. In this case the angular component of vorticity in the cylindrical coordinates alone controls blow-up of the higher Sobolev norms of the velocity.

## 1. Introduction

The Euler equations for homogeneous inviscid incompressible fluid flows in $\mathbf{R}^3$ are

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\nabla p \quad \text{in } \mathbf{R}^3 \times \mathbf{R}_+ , \tag{1}$$

$$\nabla \cdot v = 0 \quad \text{in } \mathbf{R}^3 \times \mathbf{R}_+ , \tag{2}$$

$$v( \cdot ,0) = v_0 \quad \text{in } \mathbf{R}^3 . \tag{3}$$

Here $v = (v_1(x,t), v_2(x,t), v_3(x,t))$ is the velocity of the fluid flow, $p = p(x,t)$ is the scalar pressure, and $v_0$ is the initial velocity satisfying $\nabla \cdot v_0 = 0$. Taking the curl of (1), we obtain the equation for the vorticity $\omega = \nabla \times v$,

$$\frac{\partial \omega}{\partial t} + v \cdot \nabla\omega = \omega \cdot \nabla v . \tag{4}$$

For the existence of local in time smooth solutions we have the following result by Kato [3]: Suppose an initial velocity field $v_0 \in V^m$, $m \geqq 3$, is given. Then, there exists $T_0 = T_0(\|v_0\|_{H^3})$ such that the system of equations (1)–(3) has the unique solution

$$v \in C([0,T]; V^m) \cap C^1([0,T]; V^{m-1}) \tag{5}$$

for all $T \in (0, T_0)$, where we used the function space

$$V^m = \{v \in H^m(\mathbf{R}^3) \,|\, \nabla \cdot v = 0\} .$$