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Global Spherically Symmetric Solutions to the Equations of a Viscous Polytropic Ideal Gas in an Exterior Domain

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Dedicated to Professor Rolf Leis on the occasion of his 65th birthday

Abstract: We consider the equations of a viscous polytropic ideal gas in the domain exterior to a ball in $\mathbb{R}^n$ $(n = 2$ or $3)$ and prove the global existence of spherically symmetric smooth solutions for (large) initial data with spherical symmetry. The large-time behavior of the solutions is also discussed. To prove the existence we first study an approximate problem in a bounded annular domain and then obtain a priori estimates independent of the boundedness of the annular domain. Letting the diameter of the annular domain tend to infinity, we get a global spherically symmetric solution as the limit.

1. Introduction

The motion of a viscous polytropic ideal gas in $\mathbb{R}^n$ $(n = 2$ or $3)$ is described by the following equations in Eulerian coordinates (cf. [4, 25])

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) = 0\,,$$

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] = \mu\Delta\mathbf{v} + (\lambda+\mu)\nabla(\operatorname{div}\mathbf{v}) - R\nabla(\rho\theta)\,,$$

$$c_V\rho\left[\frac{\partial \theta}{\partial t} + (\mathbf{v}\cdot\nabla)\theta\right] = \kappa_Q\Delta\theta - R\rho\theta(\operatorname{div}\mathbf{v}) + \lambda(\operatorname{div}\mathbf{v})^2 + 2\mu D\cdot D\,. \qquad (1.1)$$

Here ρ, θ, and $\mathbf{v} = (v_1, \dots, v_n)$ are the density, the absolute temperature and the velocity respectively, R, c_V and κ_Q are positive constants; λ and μ are the constant viscosity coefficients, $\mu > 0$, $\lambda + 2\mu/n \geqq 0$; $D = D(\mathbf{v})$ is the deformation tensor,

$$D_{ij} := \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \quad \text{and} \quad D\cdot D := \sum_{i,j=1}^{n} D_{ij}^2\,.$$

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