Commun. Math. Phys. 178, 311 – 338 (1996)

Communications in
Mathematical
Physics

Integrability and Huygens' Principle on Symmetric Spaces

Oleg A. Chalykh, Alexander P. Veselov ★

Department of Mathematics and Mechanics, Moscow State University, 119899 Moscow, Russia

Received: 9 June 1995 / Accepted: 17 July 1995

Abstract: The explicit formulas for fundamental solutions of the modified wave equations on certain symmetric spaces are found. These symmetric spaces have the following characteristic property: all multiplicities of their restricted roots are even. As a corollary in the odd-dimensional case one has that the Huygens' principle in Hadamard's sense for these equations is fulfilled. We consider also the heat and Laplace equations on such a symmetric space and give explicitly the corresponding fundamental solutions – heat kernel and Green's function. This continues our previous investigations [16] of the spherical functions on the same symmetric spaces based on the fact that the radial part of the Laplace–Beltrami operator on such a space is related to the algebraically integrable case of the generalised Calogero–Sutherland–Moser quantum system. In the last section of this paper we apply the methods of Heckman and Opdam to extend our results to some other symmetric spaces, in particular to complex and quaternian grassmannians.

Introduction

It is well-known that the behaviour of the solutions of the wave equation on the plane and in the space is rather different. A pointwise disturbance in $\mathbf{R}^3$ generates a pure spherical wave, while on the plane the disturbed domain is the whole disc. In the first case we say that *Huygens' principle* holds (in Hadamard's sense), in the latter we have *the wave diffusion* (see e.g. [1]).

More precisely, Huygens' principle (HP) for a second order hyperbolic equation means that the fundamental solution of the corresponding Cauchy problem is located on the characteristic conoid. The problem of finding all hyperbolic equations with HP is known as *Hadamard's problem.* In his fundamental lectures on the Cauchy problem [2] in Yale University in 1923 Hadamard found the criterion for the validity of HP and as a result he proved that HP is impossible in the even-dimensional

★ Present address: Department of Mathematical Sciences, Loughborough University of Technology, Loughborough, Leicestershire, LE11 3TU, United Kingdom
e-mail address: A.P.Veselov@lut.ac.uk