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An Anomaly Associated with 4-Dimensional Quantum Gravity

Thomas Branson

Department of Mathematics, University of Iowa, Iowa City, IA 52242, USA.
(E-mail: branson@math.uiowa.edu)

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Abstract: We compute the functional determinant quotient $(\det P_h)/(\det P_g)$ for the Paneitz operator P in conformally related Riemannian metrics g, h, and discuss related positivity questions.

1. Introduction

In 1983, Paneitz introduced a fourth-order differential operator invariant P of conformal manifolds, which has in many aspects proved to be the analogue of the two-dimensional scalar Laplacian in four-dimensional conformal theory [P]. The original application of this operator was to conformally invariant gauge-fixing for the Maxwell equations, but it has played important roles in the later studies [B1–3, BCY, BØ3, CY, ES1–2]. In [C, IV.4.γ], the functional determinant of P is found to define an anomaly associated to a quantum gravitational action defined using the Wodzicki residue [Wo].

In [BØ3], it was shown that determinant quotients for a large class of operators $\mathscr{D}$ which includes P are explicitly computable in dimension 4. The quotients in question have the form $(\det A_\omega)/(\det A_0)$, where $A \in \mathscr{D}$, A_0 is the operator A evaluated in some background metric g_0, and A_ω is the same operator evaluated in a conformally related metric $g_\omega = e^{2\omega} g_0$, where ω is a C^∞ function. The underlying space is a 4-dimensional smooth manifold M equipped with a conformal structure compatible with g_0. By Schoen's solution of the Yamabe problem [S], we may assume g_0 has constant scalar curvature. The class $\mathscr{D}$ consists of formally self-adjoint operators with positive definite leading symbol, which are positive integral powers of conformally covariant differential operators. These operators can act on tensor-spinors (the square of the Dirac operator is an example), but in this paper we shall only look at scalar operators. Of course the "operators" we speak of are really functors which canonically assign an operator to each manifold of this type; an example of such a functor is provided by the Yamabe operator (conformal Laplacian). A conceptual point which has been crucial in the later works [B3, BCY, CY] (see also [Be]) is the decision in [BØ3] to use a basis for the invariants appearing in the operator