Commun. Math. Phys. 178, 237–264 (1996)

Communications in Mathematical Physics

Quantum Affine Algebras and Deformations of the Virasoro and $\mathcal{W}$-Algebras

Edward Frenkel[1], **Nikolai Reshetikhin**[2]

[1] Department of Mathematics, Harvard University, Cambridge, MA 02138, USA
[2] Department of Mathematics, University of California, Berkeley, CA 94720, USA

Received: 1 June 1995

Abstract: Using the Wakimoto realization of quantum affine algebras we define new Poisson algebras, which are q-deformations of the classical $\mathcal{W}$-algebras. We also define their free field realizations, i.e. homomorphisms into some Heisenberg–Poisson algebras. The formulas for these homomorphisms coincide with formulas for spectra of transfer-matrices in the corresponding quantum integrable models derived by the Bethe–Ansatz method.

1. Introduction

1.1. In this paper we generalize some results concerning affine Kac–Moody algebras at the critical level to the corresponding quantized universal enveloping algebras. Here is a short description of these results for the affine algebras.

(i) Let $\widetilde{U}(\widehat{\mathfrak{g}})_{\mathrm{cr}}$ be a completion of the universal enveloping algebra of an affine algebra $\widehat{\mathfrak{g}}$ at the critical level $-h^\vee$ (the precise definition is given in Sect. 2). This algebra possesses a large center $Z(\widehat{\mathfrak{g}})$, which has a natural Poisson structure. B. Feigin and E. Frenkel have shown that $Z(\widehat{\mathfrak{g}})$ is isomorphic to the classical $\mathcal{W}$-algebra $\mathcal{W}(\mathfrak{g}^L)$ associated to the simple Lie algebra $\mathfrak{g}^L$, which is Langlands dual to $\mathfrak{g}$ [1].

(ii) The $\mathcal{W}$-algebra $\mathcal{W}(\mathfrak{g}^L)$ consists of functionals on a certain Poisson manifold $\mathscr{C}(\mathfrak{g}^L)$ obtained by the Drinfeld–Sokolov hamiltonian reduction [2] from a hyperplane in the dual space to the affine algebra $\widehat{\mathfrak{g}^L}$. Elements of $\mathscr{C}(\mathfrak{g}^L)$, called $\mathfrak{g}^L$-opers in [3], can be considered as connections on a certain G^L-bundle over the circle with some extra structure. To a $\mathfrak{g}^L$-oper one can attach a $\widehat{\mathfrak{g}}$-module, on which the center acts according to the corresponding character. These $\widehat{\mathfrak{g}}$-modules can be considered as analogues of admissible representations of a simple group over a local non-archimedian field. They can be used in carrying out the geometric Langlands correspondence proposed by A. Beilinson and V. Drinfeld [3].

Partially supported by NSF grants DMS-9205303 and DMS-9296120.