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Operator Coproduct-Realization of Quantum Group Transformations in Two-Dimensional Gravity I

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Abstract: A simple connection between the universal R matrix of $U_q(sl(2))$ (for spins $\frac{1}{2}$ and J) and the required form of the coproduct action of the Hilbert space generators of the quantum group symmetry is put forward. This leads us to an explicit operator realization of the coproduct action on the covariant operators. It allows us to derive the expected quantum group covariance of the fusion and braiding matrices, although it is of a new type: the generators depend upon worldsheet variables, and obey a new central extension of the $U_q(sl(2))$ algebra realized by (what we call) fixed point commutation relations. This is explained by showing on a general ground that the link between the algebra of field transformations and that of the coproduct generators is much weaker than previously thought. The central charges of our extended $U_q(sl(2))$ algebra, which includes the Liouville zero-mode momentum in a non-trivial way, are related to Virasoro-descendants of unity. We also show how our approach can be used to derive the Hopf algebra structure of the extended quantum-group symmetry $U_q(sl(2)) \odot \hat{U_q}(sl(2))$ related to the presence of both of the screening charges of 2D gravity.

1. Introduction

The quantum group structure of two-dimensional gravity in the conformal gauge has led to striking developments [1–13], by allowing us to derive general formulae for the fusion and braiding coefficients of the operator product algebra (OPA) in terms of quantum group symbols of $U_q(sl(2))$. Moreover, there exists [1, 2, 3, 5] a covariant basis of holomorphic operators, where there is a natural quantum group action which is a symmetry of the OPA. However, this characterization of the quantum group symmetry is somewhat implicit, as so far we do not have an explicit construction of the $U_q(sl(2))$ generators as operators on the Hilbert space of

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