Commun. Math. Phys. 178, 115 – 134 (1996)

Communications in Mathematical Physics

Stable Singularities in String Theory

Paul S. Aspinwall[1], **David R. Morrison**[2]
(with an Appendix by Mark Gross[3,★]**)**

[1] F.R. Newman Lab. of Nuclear Studies, Cornell University, Ithaca, NY 14853, USA.
E-mail: psa@hepth.cornell.edu
[2] Department of Mathematics, Box 90320, Duke University, Durham, NC 27708, USA.
E-mail: drm@math.duke.edu
[3] Department of Mathematics, Cornell University, Ithaca, NY 14853, USA.
E-mail: mgross@math.cornell.edu

Received: 10 April 1995/Accepted: 3 August 1995

Abstract: We study a topological obstruction of a very stringy nature concerned with deforming the target space of an $N = 2$ non-linear σ-model. This target space has a singularity which may be smoothed away according to the conventional rules of geometry, but when one studies the associated conformal field theory one sees that such a deformation is not possible without a discontinuous change in some of the correlation functions. This obstruction appears to come from torsion in the homology of the target space (which is seen by deforming the theory by an irrelevant operator). We discuss the link between this phenomenon and orbifolds with discrete torsion as studied by Vafa and Witten.

1. Introduction

A very interesting aspect of string theory is the way in which space-time is described. In physics, thanks to the success of general relativity, we are accustomed to picturing space-time as being a manifold equipped with a metric. The physics of space-time is then described in terms of this metric. Such a picture has some potential shortcomings. In particular we may wish to consider some space-time which is not smooth and thus may not admit a metric in the conventional sense. One way to treat such a space may be as a limit of a sequence of smooth manifolds which converges to the desired space. Thus the "metric" on the singular space is approximated by this sequence of smooth metrics.

While such a picture appears natural from a viewpoint of general relativity it may be that it is not so natural from a string theory point of view. In this paper we illustrate precisely this point by considering a singular space which classically appears as the limit of a sequence of smooth manifolds and then showing that a string theory on the singular space cannot be deformed into a string theory on any of the smooth manifolds which approximate it.

★ Supported in part by NSF grant DMS-9400873.